Tutorial 3

1. Find the eigenvalue and eigenvectors of matrix A:

$$A = \begin{pmatrix} 1 & -2 & 2 \\ -2 & -2 & 4 \\ 2 & 4 & -2 \end{pmatrix}$$

2. Find inverse matrix A^{-1}:

$$A = \begin{pmatrix} 0 & 2 & -1 \\ 1 & 1 & 2 \\ -1 & -1 & -1 \end{pmatrix}$$

3. $X(t)$ is wide-sense stationary, $Y(t) = X(t) \cdot \cos(wt + \theta)$: w is a constant and θ is uniform distributed random variable between $[-\pi, +\pi]$, θ and X(t) are independent. Is Y(t) wide-sense stationary?

4. For any given scalar random variables X, Y and Z, prove the following properties of covariance.
   a. Show that $\mathrm{cov}(X, Y+Z) = \mathrm{cov}(X, Y) + \mathrm{cov}(X+Z)$
   b. Show that $\mathrm{cov}(X, Y) = \mathrm{cov}(X, E[Y|X])$
   c. Suppose that $E[Y|X] = \alpha + \beta X$ for some constants α, β. Using the result in part b above, show that $\beta = \dfrac{\mathrm{cov}(X, Y)}{\mathrm{var}(x)}$

5. Consider the stationary n-vector process $\{x_k\}$ generated by the equations

$$x_{k+1} = Ax_k + be_k$$

Here, A is a given $n \times n$ matrix and b is a given n-vector. $\{e_k\}$ is a sequence of zero mean. uncorrelated random variables, each with variance σ^2. Write

$$R_x(0) = E[x_k x_k^T]$$

Show that $R_x(x)$ satisfies the matrix equation

$$R_x(0) = A \cdot R_x(0) \cdot A^T + \sigma^2 bb^T$$